

Roma, November 11, 2009.

**Report on the  
Carlo Giannini Research Fellowship in Econometrics 2007/09**

Please find below a resume of my activity as Carlo Giannini Fellow 2007/09.

**Research**

During these two years I have been working in close collaboration with Prof. Paolo Paruolo, JRC and Università dell'Insubria, Varese, on the representation theory for vector autoregressive processes (VAR); this has led to three new papers (n.1, 2, and 3 in the list).

In **paper n.1** we present an additive decomposition of the moving average representation of VAR processes into cyclical components. We give a definition of cyclical component that encompasses seasonal components as well as (stationary) long-run and short-run fluctuations. Each cyclical component is univariate and is characterized by a spectrum with peak at a given spectral frequency. The representation is unique and it provides explicit formulae for the the (dynamic) loadings of the variables onto the different cyclical components. We discuss relations with existing definitions of stochastic cycles. Finally we discuss how to obtain cancelations of the cyclical components by linear combinations and/or by filtering of the observed series. This paper is still **work in progress**.

In **paper n.2** we present duality results for subspaces involved in the inversion of square matrix polynomials with real coefficients. These results characterize left- and right- null spaces of the inverse in terms of subspaces associated with the original matrix polynomial. In particular, we introduce a procedure which we call polynomial rank factorization that provides a characterization of the set of reduced rank restrictions that are satisfied by the coefficients of a square polynomial whose inverse function has a pole of given order at a specific point. This paper is still **work in progress**.

**Paper n.3** has been revised and resubmitted to **Journal of Econometrics**; it presents necessary and sufficient conditions for the existence of common cyclical features in VAR processes integrated of order 0, 1, 2, where the common cyclical features correspond to codependence (CD), serial correlation common features (CS), or commonality in the final equations (CE). The results are based on polynomial rank factorizations of the reversed AR polynomial around its poles. All processes with CS structures are found to have CE structures and vice versa; the presence of CD structures, instead, implies the presence both CS and CE structures but not vice versa. Characterizations of the CS, CE, CD linear combinations are given in terms of linear subspaces defined in the polynomial rank factorizations.

The focus of my research has been centered on the papers above; on top of that, I have concluded three older projects (n.4, 5, and 6 in the list).

With Prof. Javier Ordóñez, Universitat Jaume I, Castellon de la Plana, Spain, I have applied Smooth Threshold Autoregressive (STAR) models to the dynamics of European unemployment rates; **paper n.4** has been **submitted** and **paper n.5** has been published in **Economic Letters**.

**Paper n.6** has been accepted for publication in **Econometric Theory**; it extends the representation theory of the autoregressive model in the fractional lag operator of Johansen (ET 2009). A recursive algorithm for the characterization of cofractional relations and the corresponding adjustment coefficients is given and it is shown under which condition the solution of the model is fractional of order  $d$  and displays cofractional relations of order  $d - b$  and polynomial cofractional relations of order  $d - 2b, \dots, d - cb \geq 0$  for integer  $c$ ; the cofractional relations and the corresponding moving average representation are characterized

in terms of the autoregressive coefficients by the same algorithm. For  $c = 1$  and  $c = 2$  we find the results of Johansen (ET 2009).

**Papers** (from the most recent backwards)

1. with P. Paruolo, *Stochastic cycles in VAR processes*, mimeo 2009.
2. with P. Paruolo, *Subspace duality in the inversion of square matrix polynomials*, mimeo 2009.
3. with P. Paruolo, *A characterization of VAR processes with common cyclical features*, 2008, revised and resubmitted to **Journal of Econometrics**.
4. with J. Ordóñez, *Multiple equilibria in Spanish unemployment*, mimeo 2008, submitted.
5. with J. Ordóñez, *Common smooth transition trend-stationarity in European unemployment*, 2008, **Economics Letters** 101(2), p.106-109.
6. *A representation theory for polynomial cofractionality in vector autoregressive models*, 2010, forthcoming in **Econometric Theory**.

**Conference and seminar presentations** (from the most recent backwards)

1. *A characterization of VAR processes with common cyclical features*, EIEF, Roma, November 2009.
2. ———, *Econometrics, Time Series Analysis and Systems Theory*, Conference in Honor of Manfred Deistler, Wien, June 2009.
3. ———, *Italian Congress of Econometrics and Empirical Economics (ICEEE)*, Ancona, January 2009.
4. ———, *Joint Research Centre of the European Commission*, Ispra, September 2008.
5. ———, *Università di Padova*, September 2008.
6. ———, *Conference on Factor Structures for Panel and Multivariate Time Series Data*, Maastricht, September 2008.
7. ———, *University of Copenhagen*, September 2008.
8. ———, *NBER-NSF Time Series Conference*, Århus, September 2008.
9. *A representation theory for polynomial cofractionality in vector autoregressive models*, European Econometric Society Meeting (ESEM), Milano, August 2008.

**Referee activity**

Econometric Theory (1), Economics E-journal (1).

**Teaching**

2008. 14 hours of “Econometrics of Financial Markets” with Paolo Paruolo (Varese), 20 hours of “Econometrics I” at Scuola di Dottorato in Economia della Sapienza (Roma) and 12 hours of “Cointegration” at CIDE Summer School in Econometrics (Bertinoro).
2009. 20 hours of “Econometrics I” at Scuola di Dottorato in Economia della Sapienza (Roma).

**Future projects**

Complement the representation theory with the development of the inference part.

**Thanks**

I wish to thank CIdE, Associazione Carlo Giannini and the Bank of Italy for giving me the opportunity of spending two years of full time research with Paolo Paruolo; this has been a highly challenging and rewarding period, during which I have really learnt a lot. Without the generosity of the Carlo Giannini Fellowship and the guidance of Paolo Paruolo this would have been impossible. I also would like to thank Dipartimento di Economia, Università dell'Insubria, Varese, and Joint Research Centre of the European Commission, Ispra, for the hospitality. I am very grateful to Paolo Paruolo for his continuous help, support and interest and I wish the next Carlo Giannini Fellow an equally satisfactory experience and the very best achievements.

Yours sincerely

Massimo Franchi