

Carlo Cellucci, *La filosofia della matematica del Novecento*, Laterza, Bari 2007

## **Philosophy of Mathematics in the Twentieth Century**

Carlo Cellucci

*Preface*

### **I Philosophy and mathematics**

1. *The prevailing orthodoxy*

1.1. Mathematics vs. philosophy of mathematics - 1.2. Philosophy of mathematics vs. philosophical tradition

2. *Limitations of the prevailing orthodoxy*

2.1. Limitations of the autonomy of the philosophy of mathematics - 2.2. Limitations of the polemic against the philosophical tradition

### **II The philosophy of mathematics of yesterday**

1. *Frege*

1.1. Frege's motivations - 1.2. Frege's program - 1.3. Frege's view of logic - 1.4. Frege's debt to Kant and Leibniz - 1.5. Deviations from Leibniz - 1.6. Frege's arguments against Kant - 1.7. Hume's principle - 1.8. Julius Caesar problem - 1.9. The riddle of defining the extension of a concept - 1.10. The acme of Frege's program - 1.11. Julius Caesar problem again - 1.12. Russell's paradox - 1.13. The fall of Frege's program - 1.14. Frege's final reaction

2. *Hilbert*

2.1. Hilbert's motivations - 2.2. Finitary and infinitary mathematics - 2.3. Hilbert's aim - 2.4. The conservation program - 2.5. The consistency program - 2.6. Adequacy of the consistency program - 2.7. Hilbert's debt to Kant - 2.8. Deviations from Kant - 2.9. Expectations concerning the feasibility of the programs - 2.10. The fall of the consistency program - 2.11. Detlefsen's objection - 2.12. The fall of the conservation program - 2.13. Inadequacy of consistency - 2.14. Kant's reasons - 2.15. Hilbert's final reaction

### 3. *Brouwer*

3.1. Brouwer's motivations - 3.2. Brouwer's program - 3.3. The rejection of the excluded middle - 3.4. The intuitionistic notion of proof - 3.5. The two acts of intuitionism - 3.6. Brouwer's debt to Kant - 3.7. Deviations from Kant - 3.8. The intuitionistic continuum - 3.9. The continuity theorem - 3.10. The limitations of Brouwer's program - 3.11. Brouwer's aestheticism - 3.12. The fall of Brouwer's program

### 4. *Conclusions over the philosophy of mathematics of yesterday*

## **III The philosophy of mathematics of today**

### 1. *Two missing reactions*

1.1. A mathematical missing reaction - 1.2. A philosophical missing reaction

### 2. *The philosophical conceptions of the second half of the twentieth century*

2.1. Neologicism - 2.2. Platonism - 2.3. Implicationism - 2.4. Structuralism - 2.5. Fictionalism - 2.6. Internalism - 2.7. Constructivism - 2.8. Conjecturalism - 2.9. Empiricism - 2.10. Cognitivism

### 3. *Conclusions over the philosophy of mathematics of today*

## **IV The philosophy of mathematics of tomorrow**

### 1. *Characters of the philosophy of mathematics of tomorrow*

1.1. The need for a new beginning - 1.2. Non-autonomy of the philosophy of mathematics - 1.3. Relationship with mathematics - 1.4. Limitations of the question of the foundation of mathematics - 1.5. Centrality of the discovery problem

### 2. *The image of mathematics*

2.1. A requirement for the realization of the program - 2.2. Mathematics and experience - 2.3. Mathematics and problem solving - 2.4. Mathematics and evolution - 2.5. Mathematics and cognitive architectures - 2.6. Mathematics and historical development - 2.7. Mathematics and truth

## **V Gödel's incompleteness theorems**

## 1. *First order logic*

1.1. First order languages - 1.2. Axioms and rules for first order logic - 1.3. Models for first order languages - 1.4. Consistency - 1.5. Soundness and completeness - 1.6. Isomorphism of models - 1.7. First order theories

## 2. *Primitive recursive arithmetic*

2.1. Primitive recursive functions - 2.2. The theory **PRA** - 2.3. Some elementary properties of **PRA** - 2.4. Sufficiently powerful theories

## 3. *Codification*

3.1. Gödel numbers - 3.2. RE-sets - 3.3. RE-theories - 3.4. Some useful primitive recursive functions

## 4. *The incompleteness theorems*

4.1. The fixed point theorem - 4.2. Gödel's first incompleteness theorem - 4.3. Corollaries of Gödel's first incompleteness theorem Gödel - 4.4. Gödel's second incompleteness theorem - 4.5. Relevance of the expression of consistency - 4.6. Rosser's incompleteness theorem - 4.7. Gödel's third incompleteness theorem - 4.8. Comparison between Gödel's incompleteness theorems - 4.9. Extension to other theories

## 5. *Other limitative results*

5.1. Tarski's undefinability theorems - 5.2. Undecidability theorem - 5.3. Church's theorem - 5.4. Extension to other theories

## 6. *Second order theories*

6.1. Second order languages - 6.2. Axioms and rules for second order logic - 6.3. Models for second order languages - 6.4. Isomorphism of models - 6.5. Second order theories

## 7. *Second order Peano arithmetic*

7.1. The theory  $\mathbf{PA}^2$  - 7.2. Gödel's incompleteness theorems for  $\mathbf{PA}^2$  - 7.3. Other limitative results for  $\mathbf{PA}^2$  - 7.4. Categoricity of  $\mathbf{PA}^2$  - 7.5. Relation between recursive enumerability and arithmeticity - 7.6. Strong incompleteness of second order logic - 7.7. Non-arithmeticity of logical consequences of  $\mathbf{PA}^2$  - 7.8. Weak models for second order languages - 7.9. Non-standard models of  $\mathbf{PA}^2$

## *Guide to Further Readings*

## *References*

*Index of Names*