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CLASSIFYING AND JUSTIFYING INFERENCE RULES

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SUMMARY: It is a widespread view that inferences can be either deductive, that is, necessarily truth preserving, or ampliative, that is, not necessarily truth preserving. This view is inadequate because there are inferences, such as the abductive ones, that are neither ampliative nor truth preserving. In this paper an alternative classification of inferences is proposed, as well as a justification of deductive, non-deductive and abductive inferences which takes into account their role in knowledge, distinguishing their justification from their usefulness. It is argued that the justification of deductive, non-deductive and abductive inferences raises similar problems and is to be approached much in the same way.

KEYWORDS: Inference, deductive, non-deductive, classification, justification

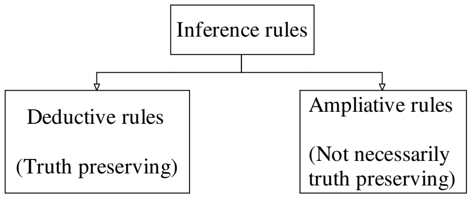
1. *The Standard Classification of Inference Rules*

Inference rules are usually distinguished into deductive and non-deductive. But are all inference rules either deductive or non-deductive? This is the question of the classification of inference rules.

In addition to this question, there is the question of what justifies inference rules and what their role is in knowledge. This is the question of the justification of inference rules and the characterization of their role in knowledge.

First we consider the question of the classification of inference rules. This is a basic logic task. Peirce even claims that “the chief business of the logician is to classify arguments” (Peirce 1931–58, 2.619).

What can be termed the standard classification of inference rules, or equivalently, of inferences, is stated by Hintikka and Sandu as follows: “Inferences can be either deductive, that is, necessarily truth preserving, or ampliative, that is, not necessarily truth preserving” (Hintikka and Sandu 2007, p. 13).



This classification, however, is inadequate because, as we will see, there are inference rules which are neither deductive nor ampliative.

2. *Abduction*

It is widely held that one of the most important means of obtaining hypotheses is abduction, that is, the rule:

$$(\mathrm{ABD})\,\frac{B \to A \quad A}{B}\,.$$

For example, Josephson states that (ABD) includes "the whole process of generation, criticism, and possible acceptance of explanatory hypotheses" (Josephson 1994, p. 9).

Of course, (ABD) is not necessarily truth preserving because, if A is true and B is false, then $B \to A$ is true, so both premises of (ABD) are true but the conclusion B is false.

Since (ABD) is not necessarily truth preserving, in terms of the standard classification of inference rules one might conclude that (ABD) is ampliative.

In fact Josephson states that (ABD) "generates new information that was not previously encoded" in its premises "at all" (ibid., p. 13).

But, that (ABD) is ampliative is in conflict with the fact that conclusion B is a subformula of the major premise $B \to A$, and so is already contained in it.

Therefore, contrary to Josephson's claim, (ABD) does not generate new information that was not previously encoded in its premises. New information is not generated by (ABD) but rather by the process that yields its major premise $B \to A$, thus it is generated before (ABD). Therefore (ABD) is non-ampliative.

Since (ABD), on the one hand, like deductive rules, is non-ampliative, and, on the other hand, like non-deductive rules, is not necessarily truth preserving, it provides a counterexample to the standard classification of inference rules.

3. *Peirce on the Status of Abduction*

Even Peirce, who first stated (ABD), acknowledges that (ABD) is non-ampliative.

In fact, Peirce states (ABD) as follows, except that, for uniformity with the above statement of (ABD), we replace the letters *C* and *A*, originally used by Peirce, by the letters *A* and *B*, respectively: "The surprising fact, *A*, is observed; But if *B* were true, *A* would be a matter of course," that is, $B \rightarrow A$; "Hence, there is reason to suspect that *B* is true" (Peirce 1931–58, 5.189). Then he explains: "Thus, *B* cannot be abductively inferred, or if you prefer the expression, cannot be abductively conjectured until its entire content is already present in the premise, 'If *B* were true, *A* would be a matter of course'" (ibid.). This provides "an argument to show that quite new conceptions cannot be obtained from abduction" (ibid., 5.190).

Thus Peirce admits that, in (ABD), the entire content of the conclusion *B* is already present in the major premise $B \rightarrow A$, and hence (ABD) is non-ampliative.

Peirce also admits that new information is not generated by (ABD) but rather by the process that yields the major premise $B \rightarrow A$. In his view, however, such process is a non-inferential one because it is based on intuition.

In fact, Peirce states that the suggestion of the major premise, $B \rightarrow A$, "comes to us like a flash. It is an act of insight, although of extremely fallible insight" (ibid., 5.181). The "different elements of the hypothesis were in our mind before; but it is the idea of putting" them "together which flashes the new suggestion before our contemplation" (ibid.). This suggestion is "the result of a process, although of a process" which is "not controllable and therefore not fully conscious," rather, it is a "subconscious process" (ibid.).

This process cannot be subjected to logical analysis. A logical analysis would be "terminated in what that analysis would represent as an abductive inference, resting on the result of a similar process, which a similar logical analysis would represent to be terminated by a similar abductive inference, and so on *ad infinitum*" (ibid.). Therefore a logical analysis would fail for the very same reason Achilles's chase for the Tortoise fails. Just "as Achilles does not have to make the series of distinct endeavors which he is represented as making," so this process of forming the judgment $B \rightarrow A$, being "subconscious and so not amenable to logical criticism, does not have to make separate acts of inference" (ibid.).

Thus Peirce distinguishes between the act of insight yielding the major premise $B \rightarrow A$, which is creative and subconscious, and (ABD), which is non-creative and conscious. (ABD) is a rule of inference and, according to Peirce, in an inference "we should be conscious, not only of the conclusion, and of our deliberate approval of it, but also of its being the result of the premises" (Peirce 1992–1998, II, p. 348). An "inference is essentially deliberate, and self-controlled" (Peirce 1931–58, 5.108). Then (ABD) cannot be creative, since "self-control of any kind is purely inhibitory. It originates nothing" (ibid., 5.194).

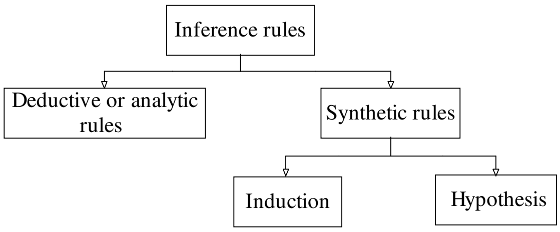
Peirce's claim that the process of forming the judgment $B \rightarrow A$ must be subconscious and based on an act of insight depends on his argument that such process cannot be subjected to logical analysis. The analytic method provides a counterexample to Peirce's argument since it gives a logical analysis of the process of hypothesis formation in terms of conscious inferences (see Cellucci 2008a,b).

In any case, what is important is that Peirce admits that (ABD) is non-ampliative.

4. *Peirce's Classification of Inference Rules*

(ABD) provides a counterexample not only to the standard classification of inference rules, but also to Peirce's own classification.

According to Peirce, inference rules may be divided into deductive or analytic, namely non-ampliative, and synthetic, namely ampliative. In turn, synthetic rules "may be divided into induction and hypothesis" (Peirce 1931–58, 2.642). Thus Peirce classifies all inference rules as follows:



By 'hypothesis' Peirce here means what he will later on call 'abduction.' This appears from his example of hypothesis: "All the beans from this bag are white" and "These beans are white," therefore "These beans are from this bag" (ibid.). This is an instance of (ABD).

(ABD) provides a counterexample to Peirce's classification of inference rules since (ABD) is non-ampliative, thus non-synthetic. Peirce's admission that (ABD) is non-ampliative also contrasts with his classification of inference rules.

The explanation of this apparent inconsistency is that Peirce's classification of inference rules and his acknowledgment that (ABD) is non-ampliative belong to different phases of his thought.

Another problem with Peirce's classification of inference rules is that, while originally he does not sharply distinguish between the roles of induction and (ABD), later on he considers (ABD) as a means of introducing new hypotheses, and induction only as a means of verifying them.

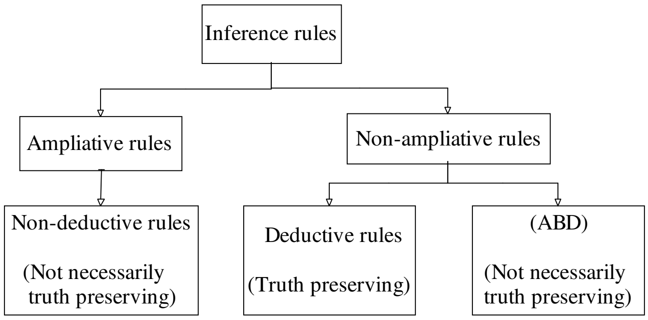
Indeed, he states that (ABD) "furnishes the reasoner with the problematic theory which induction verifies" (Peirce 1931–58, 2.776). Induction is "the experimental testing of a theory," so it "never can originate any idea whatever" (ibid., 5.145).

But, to consider induction as a means of verifying hypotheses, seems hardly plausible. For scientific hypotheses are typically universal, so no finite number of experiments could verify them.

5. *An Alternative Classification of Inference Rules*

Since (ABD) provides a counterexample to the standard classification of inference rules, an alternative classification is needed.

To that end, we divide all inference rules into ampliative and non-ampliative. Ampliative rules consist of non-deductive rules and are not necessarily truth preserving. Non-ampliative rules are divided into deductive rules and (ABD), where the former are truth preserving while the latter is not necessarily truth preserving.



From this it becomes apparent why the standard classification of inference rules is inadequate: (ABD) is both non-ampliative and not necessarily truth preserving. The only adequate classification of inference rules is in terms of ampliativity rather than truth preservation.

That non-deductive rules are not necessarily truth preserving is a consequence of their being ampliative. On the other hand, deductive rules are truth preserving because they sacrifice ampliativity. Their conclusion contains no more, in fact usually less than was implicitly contained in the premises. The conclusion is only a reformulation of all or part of the content of the premises.

(ABD) belongs to a different category because, on the one hand, like deductive rules, it is non-ampliative, and on the other hand, unlike them, it is not necessarily truth preserving.

It is an open question whether, in addition to (ABD), there are other interesting inference rules that are both non-ampliative and not necessarily truth preserving.

6. *Failures in Justifying Deductive Rules*

After discussing the question of the classification of inference rules, we now discuss that of the justification of inference rules and the characterization of their roles in knowledge. First we consider deductive rules.

Several justifications of deductive rules have been proposed. We will consider two especially significant ones: the truth-functional and the inferential justification.

(1) According to the truth-functional justification, deductive rules are justified because they are truth preserving.

For example, Frege states that the justification of modus ponens (MP) is: "From the propositions '$\vdash \Delta \rightarrow \Gamma$' and '$\vdash \Delta$' we may infer '$\vdash \Gamma$'; for if Γ were not the True, then since Δ is the True, '$\Delta \rightarrow \Gamma$' would be the False" (Frege 1964, p. 57). Here Frege's bidimensional notation has been replaced by the current unidimensional notation.

Such justification of (MP), however, is circular because it is of the following kind. Suppose that $A \rightarrow B$ and A are true. Now, if $A \rightarrow B$ is true, then, if A is true, then B is true. Since $A \rightarrow B$ and A are true, from this, by two applications of (MP), we conclude that B is true. Since this justification of (MP) uses (MP), it is circular.

Against the conclusion that the truth-functional justification of deductive rules is circular, two objections might be raised.

(i) The circularity involved in the truth-functional justification of deductive rules is not premise-circularity but rule-circularity. Premise-circularity occurs when the fact that a deductive rule is truth preserving is used as a premise in the justification of that rule. Rule-circularity occurs when a deductive rule is used in the course of the justification of that rule. Now, premise-circularity is harmful, because a premise-circular justification of a deductive rule has no force for anyone who does not already accept that such deductive rule is truth preserving. Rule-circularity is harmless, because a rule-circular justification of a deductive rule is not intended to persuade anyone of the truth of the conclusion of the deductive rule, but only to explain why the conclusion is true.

This objection, however, is untenable. For, if rule-circularity is allowed, then one can give a truth-functional justification not only of (MP), which is truth preserving, but also of (ABD), which is not necessarily truth preserving.

The truth-functional justification of (ABD) is as follows. Suppose that $B \rightarrow A$ and A are true. Now, $B \rightarrow ((B \rightarrow A) \rightarrow A)$ is true. Moreover, if A is true, then $(B \rightarrow A) \rightarrow A$ is true. Since $B \rightarrow ((B \rightarrow A) \rightarrow A)$ and $(B \rightarrow A) \rightarrow A$ are true, by an application of (ABD) we conclude that B is true.

It is no use to say that this is no genuine justification of (ABD) because, while the truth-functional justification of (MP) uses (MP), which is truth preserving, this justification of (ABD) uses (ABD), which is not necessarily truth preserving. For to justify that (MP) is truth preserving is exactly the question at issue here.

(ii) The truth-functional justification of (MP) involves use of (MP) in the metalanguage, while the (MP) being justified concerns the object language. So the rule being justified and the justifying rule belong to distinct levels.

This objection too is untenable. For how will we justify the (MP) of the metalanguage? By demonstrating that it is truth preserving, using (MP) in the meta-metalanguage. The same question will then arise for the latter, and so on, *ad infinitum*. Thus we cannot argue that (MP) is truth preserving without infinite regress.

It is no use to say that we can escape this problem if we can demonstrate that the (MP) of the object-language is truth preserving by using a deductive rule other than (MP), say (R), of the metalanguage. For, how will we justify such rule (R)? Simply, by demonstrating that (R) is truth preserving, and such demonstration will use either (MP) or (R) or a different deductive rule. If it uses (MP) or (R), we will be in a situation like the previous one. If it uses a different deductive rule, such rule must be justified and, if we try to justify it, once again we will have an infinite regress.

Moreover, the objection overlooks that if, by use of (MP) in the metalanguage, we can justify use of (MP) in the object language, then, as we have seen, by use of (ABD) in the metalanguage we can justify use of (ABD) in the object language. Thus, if (MP) is justified, then (ABD) too is justified.

We conclude that the truth-functional justification of deductive rules is inadequate.

(2) According to the inferentialist justification, the introduction rules are self-justifying since they are the definitions of the symbols concerned, and the elimination rules are justified since they are the consequences of these definitions.

For example, Gentzen states that "the introductions represent, as it were, the 'definitions' of the symbols concerned, and the eliminations are no more, in the final analysis, than consequences of these definitions" (Gentzen 1969, p. 80). Accordingly, Gentzen justifies (MP) as follows: "We were able to introduce the formula $A \rightarrow B$ when there existed a derivation of B from the assumption formula A. If we then wished to use that formula by eliminating the $\rightarrow$-symbol," we "could do this precisely by inferring B directly, once A has been proved, for what $A \rightarrow B$ attests is just the existence of a derivation of B from A" (ibid., pp. 80–81).

Such justification of (MP), however, is circular. For it is of the following kind. Suppose we have deductions of $A \rightarrow B$ and A. If we have a deduction of $A \rightarrow B$, then by Gentzen's definition of $A \rightarrow B$ we have a deduction of B from A. If we have a deduction of B from A, from the latter and the given deduction of A we obtain a deduction of B. Then by two applications of (MP) we obtain a deduction of B. Since this justification of (MP) uses (MP), it is circular.

Moreover, if (MP) is justified because, if we have deductions of $A \rightarrow B$ and A, we can obtain a deduction of B, then (ABD) can be justified in the same way: If we have deductions of $B \rightarrow A$ and A, we can obtain a deduction of B.

To illustrate this point, let us consider Kepler's argument concerning Mars's orbit, which Peirce calls "the greatest piece" of abductive "reasoning ever performed" (Peirce 1931–58, 1.74). According to Peirce, Kepler's argument is an example of (ABD). For let A: There are certain irregularities in the longitudes of Mars. Let B: The orbit of Mars is elliptical. Then Kepler's argument is: The surprising fact A is observed; But, if B, then A would be a matter of course, that is, $B \rightarrow A$; Hence, there is reason to suspect that B is true.

Now, for Kepler, a sufficient condition for asserting $B \rightarrow A$ was that, from the fact A, that there are certain irregularities in the longitudes of Mars, he could deduce B, that the orbit of Mars is elliptical, since the irregularities were compatible only with a certain elliptical orbit. Indeed, Kepler argued that, to account for such irregularities, the circle "sins by excess" while a certain ellipse "sins by defect," and "between the circle and the ellipse there is nothing but another ellipse. Therefore the planet's orbit is an ellipse" (Kepler 1858–71, III, p. 400).

Thus Kepler's sufficient condition for asserting $B \rightarrow A$ was to have a deduction of B from A.

Then a justification of (ABD) can be given as follows. Suppose we have deductions of $B \rightarrow A$ and A. If we have a deduction of $B \rightarrow A$, then by Kepler's sufficient condition for asserting $B \rightarrow A$ we have a deduction of B from A. From the latter and the given deduction of A we obtain a deduction of B.

We conclude that the inferentialist justification of deductive rules is inadequate.

It could be shown that all other justifications of deductive rules that have been proposed are also inadequate (see Cellucci 2008b, ch. 26).

This is due to the fact that none of these justifications take into account the role of deductive rules in knowledge. For they are all based on the assumption that deductive rules need not appeal, for their justification, to any source external to logic, so logic is self-justifying. Their failure is a strong indication that this assumption is untenable. In fact, there is plenty of evidence that the justification of deductive rules depends on experience and on the role of deductive rules in knowledge.

7. *Validation and Vindication*

In order to give a justification we must first explain what we mean by justification. In this regard, it is convenient to use Feigl's distinction between two kinds of justification: validation and vindication. Validation is justification in the sense of an argument concerning "validating grounds" (Feigl 1971, pp. 128–9). Vindication is justification in the "sense of an argument concerning means with respect to ends" (ibid., p. 116).

To validate an inference rule is to demonstrate that it can be derived from other inference rules. To vindicate an inference rule is to demonstrate that it is appropriate to a certain end.

8. *An Inappropriate End for Vindication*

Since, to validate a deductive rule is to demonstrate that it can be derived from other deductive rules, not all deductive rules can be validated. Otherwise, there would be an infinite regress or a vicious circle. Therefore, there must be some ultimate deductive rules which cannot be validated.

For such ultimate deductive rules we can only seek vindication, that is, we can only try to demonstrate that they are appropriate to a certain end. The question is: To what end?

Feigl's answer is that they are adequate to the end of deducing "true propositions from true premises" (ibid.).

This answer, however, is unsatisfactory because, as we have seen, we cannot demonstrate without circularity that (MP) is appropriate to the end of deducing true propositions from true premises. The same applies to any other ultimate deductive rule. Therefore, we cannot demonstrate that the ultimate deductive rules are appropriate to that end.

In addition, seeking to vindicate the ultimate deductive rules with respect to the end of deducing true propositions from true premises fails to take into account the role of deductive rules in knowledge. For it overlooks that the premises on which our knowledge is based are not true but can only be plausible, that is, compatible with the existing data (see Cellucci 2008a,b). So the end of deducing true propositions from true premises is a vacuous one. Therefore the role of deductive rules in knowledge cannot be to deduce true propositions from true premises.

9. *Plausibility and Endoxa*

We have said that a premise is plausible if it is compatible with the existing data. This means that, comparing the arguments for and the arguments against the premise on the basis of the existing data, the arguments for are stronger than those against.

Plausible premises correspond to what Aristotle calls 'accepted opinions' and are used in the same sense.

Actually, Aristotle says that those things "are accepted opinions [*endoxa*] which seem so to everyone, or to the great majority, or to the wise, and among them either to all of them, or to the great majority, or to the most famous and esteemed" (Aristotle, *Topics*, A 1, 100 b 21–23).

This, however, cannot be taken to be a definition of 'accepted opinion,' because it does not tell us on what ground opinions are accepted. A definition of 'accepted opinion' is rather implicit in Aristotle's statement that, for each opinion concerning some given subject, we must "examine the arguments for and the arguments against" (ibid., Θ 14, 163 a 37–b 1).

This justifies the claim that plausible premises correspond to what Aristotle calls 'accepted opinions.'

That premises must be plausible entails that "not every premise or every problem should be counted" as a premise, and in fact "no one in his right mind would hold out as a premise what nobody thinks" (ibid., A 10, 104 a 4–6). Premises are directed "to truth and knowledge" (ibid., A 11, 104 b 2). For such reason, after saying that a premise must be something "acceptable [*endoxos*] to everyone, or to the great majority, or to the wise," and among them "either all of them, or the great majority, or to the most famous," Aristotle adds: "provided that it is not paradoxical [*paradoxos*]" (ibid., A 10, 104 a 8–11).

The proviso that a premise must not be paradoxical is essential because, for Aristotle, accepted opinions can be true. For such reason, they cannot be paradoxical. Indeed, for Aristotle, *paradoxos* is the opposite of *endoxos*.

10. *The Vindication of Deductive Rules*

As an alternative to Feigl's answer, the ultimate deductive rules are appropriate to the following end:

(A) To make explicit the content or part of the content that is implicit in the premises.

That the ultimate deductive rules can be vindicated with respect to the end (A) follows from an analysis of the premises, which shows that the conclusion of a deductive rule is implicit in them. For example, that (MP) can be vindicated with respect to the end (A) follows from the observation that the conclusion B is a subformula of the major premise $A \rightarrow B$.

The fact that the ultimate deductive rules can be vindicated with respect to the end (A) is consistent with the claim that only by taking into account the role of deductive rules in knowledge can one give a justification of such rules. For that role is just (A). Deductive rules are non-ampliative, but that does not mean that they play no useful role in knowledge. Since the conclusion makes explicit the content or part of the content that is implicit in the premises, establishing that the conclusion is plausible facilitates the comparison of the premises with experience. Therefore the end (A) agrees with the role of deductive rules in knowledge.

11. *Vindication of Deductive Rules and Usefulness*

That the ultimate deductive rules can be vindicated with respect to the end (A) does not ensure, however, that any specific deductive inference is useful.

For a deductive inference to be useful one must know that its premises and conclusion are plausible. Knowing that its premises are plausible is not enough, because deductive rules, while truth preserving, are not necessarily plausibility preserving.

For example, let us consider the conjunction introduction rule ($\wedge$I): From A and B we may infer $A \wedge B$. Suppose that A and B are both plausible. Then $A \wedge B$ is not necessarily plausible. For let A and B be two rival scientific hypotheses. In order to be rival, A and B, on the one hand, must be mutually incompatible, but, on the other hand, must both be plausible. Now, since A and B are mutually incompatible, the arguments for A will be arguments against B, and viceversa, so their conjunction, $A \wedge B$, will not be plausible. Therefore ($\wedge$I) is not plausibility preserving.

This contradicts Pólya's view that deductive rules are plausibility preserving (see Pólya 1954, II, p. 27).

Since, for a deductive inference to be useful, one must know that both its premises and conclusion are plausible, the usefulness of deductive rules essentially depends on a comparison with experience, so with something external to inference.

12. *Failures in Justifying Non-Deductive Rules*

We now discuss non-deductive rules. Several justifications have been proposed. We will consider an especially significant one: the intuitional justification.

According to the intuitional justification, non-deductive rules are justified because they ultimately rest on intuition.

For example, Kyburg states that "our justification of inductive rules must rest on an ineradicable element of inductive intuition" (Kyburg 1965, p. 276). When "we see that, if all we know about in all the world is that all the *A*'s we've seen have been *B*'s, it is rational to expect that the next *A* will be a *B*" (ibid.).

Such justification of non-deductive rules, however, is inadequate because intuition is fallible.

In view of this, some supporters of the intuitional justification of non-deductive rules replace infallible intuition with fallible intuition.

For example, Carnap states that the reasons for accepting non-deductive rules "are based upon our intuitive judgments concerning inductive validity" (Carnap 1963, p. 978). Namely, upon intuition. Admittedly, intuition is fallible because it "may on occasion lead us astray" (Carnap 1968, p. 265). But also the reasons for accepting deductive rules are based upon "intuition" (ibid., p. 266). So the "situation in inductive logic" is "not worse than that in deductive logic, but quite analogous to it" (ibid.). The fact that intuition is fallible simply means that the intuitive plausibility of a non-deductive inference "may be more or less strong; and in the course of the development of a system, there may be progress by an increase in plausibility" (ibid.). As a result, a non-deductive inference may be "replaced by one or several more plausible" non-deductive inferences, or for a given non-deductive inference "more plausible reasons are found" (ibid.).

But then the reasons for accepting non-deductive rules ultimately depend on plausibility, and plausibility is based upon argument, not upon intuition. The same applies to deductive logic, if the reasons for accepting deductive rules are based upon intuition.

Therefore, the intuitional justification of non-deductive rules is inadequate.

It could be shown that all other justifications of non-deductive rules that have been proposed are also inadequate (see Cellucci 2008b, ch. 25).

This is due to the fact that none of these justifications take into account the role of non-deductive rules in knowledge.

13. *The Vindication of Non-Deductive Rules*

To give a justification of non-deductive rules, Feigl's distinction between validation and vindication is once again useful.

Since, to validate a non-deductive rule is to demonstrate that it can be derived from other non-deductive rules, not all non-deductive rules can be validated. Otherwise, there would be an infinite regress or a vicious circle. Therefore, there must be some ultimate non-deductive rules that cannot be validated.

For such ultimate non-deductive rules we can only seek vindication, that is, we can only try to demonstrate that they are appropriate to a certain end. The question is: To what end?

The answer is that the ultimate non-deductive rules are appropriate to the following end:

(B) To discover hypotheses.

That the ultimate non-deductive rules can be vindicated with respect to end (B) follows from an analysis of the conclusion, which shows that it contains something not contained in the premises. This is essential for discovery.

That the ultimate non-deductive rules can be vindicated with respect to the end (B) is consistent with the claim that only by taking into account the role of non-deductive rules in knowledge one can give a justification of such rules. For that role is just (B). Non-deductive rules are ampliative because their conclusion essentially goes beyond the premises. For such reason they may provide a basis for discovery. Therefore, the end (B) agrees with the role of non-deductive rules in knowledge.

14. *Vindication of Non-Deductive Rules and Usefulness*

That the ultimate non-deductive rules can be vindicated with respect to the end (B) does not ensure, however, that any specific non-deductive inference is useful.

For a non-deductive inference to be useful one must know that both its premises and conclusion are plausible. To know that its premises are plausible is not enough, because obviously non-deductive rules are not necessarily plausibility preserving.

Since, for a non-deductive inference to be useful, one must know that both its premises and conclusion are plausible, the usefulness of non-deductive rules essentially depends on a comparison with experience, so with something external to inference.

15. *The Vindication of Abduction*

Since (ABD) is non-ampliative, it cannot be vindicated with respect to the end (B), to discover hypotheses. Like deductive rules, it can be vindicated only with respect to the end (A), to make explicit the content or part of the content that is implicit in the premises.

To establish that (ABD) is appropriate to the end (A), we need only observe that the conclusion B is a subformula of the major premise $B \rightarrow A$, and so is contained in it.

16. *The Asymmetry View*

It is commonly held that there is an asymmetry between deductive and non-deductive rules: While deductive rules can be justified, non-deductive rules cannot be justified.

For example, Salmon states that deductive rules are justified because, if we reflect upon a deductive rule such as (MP), we "cannot conceive the possibility of any situation in which its use would lead from true premises to false conclusions" (Salmon 1965, p. 268). So "we can find no grounds whatever for withholding the judgment" that (MP) "is truth preserving" (ibid.). On the contrary, non-deductive rules cannot be justified because "we can, without difficulty, imagine all sorts of states of affairs in which practically all – if not absolutely all – of our future inductive inferences with true premises turn out to have false conclusions" (ibid.).

Let us call this the 'asymmetry view.' However common, this view is unjustified, because the justifications of deductive and non-deductive rules raise similar problems and must be approached much in the same way. This appears from the following facts.

1) There must be both some ultimate deductive and non-deductive rules that cannot be validated. For these one can only seek vindication.

2) Both deductive and non-deductive rules can be vindicated with respect to an end that agrees with their role in knowledge. For deductive rules can be vindicated with respect to the end (A), which is their role in knowledge. Non-deductive rules can be vindicated with respect to the end (B), which is their role in knowledge.

3) The usefulness of both deductive and non-deductive rules essentially depends on a comparison with experience, so with something external to inference.

These facts suggest a symmetry, rather than an asymmetry, between deductive and non-deductive rules.

17. *Sextus Empiricus's Arguments About the Criterion of Truth*

What is the origin of the asymmetry view? It cannot be attributed to Sextus Empiricus, since he produced an argument about the criterion of truth that immediately yields an argument for the impossibility of validating all inference rules.

Indeed, Sextus Empiricus's argument is: "Those who profess to judge the truth are bound to have a criterion of truth" (Sextus Empiricus, *Against the Logicians*, I.340). Now, this criterion of truth "either is not judged upon or has been judged upon. If it is not judged upon," then it cannot be trusted, "for no matter of dispute is to be trusted without judging. If it has been judged upon, that which judges upon it, in turn, either has been judged upon or has not been judged upon, and so on *ad infinitum*" (ibid.).

If we interpret 'criterion of truth' as 'inference rule,' then Sextus Empiricus's argument becomes: Those who profess to validate an inference are bound to have some inference rule to validate it. Now this inference rule either is not validated or has been validated. If it is not validated, then it cannot be trusted, for no matter of dispute is to be trusted without being validated. If it has been validated, the inference rule used to validate it, in turn, either has been validated or has not been validated, and so on *ad infinitum*.

By this argument, it is impossible to validate all inference rules. So there must be some ultimate inference rules which cannot be validated. From this it appears that the origin of the asymmetry view cannot be attributed to Sextus Empiricus.

18. *Hume's Argument About Induction*

The origin of the asymmetry view may rather be attributed to Hume. This appears from his attitude toward induction and deduction.

According to Hume, induction cannot be justified demonstratively "since it implies no contradiction that the course of nature may change" (Hume 1975, p. 35). Induction can be justified only inductively or, as Hume says, by experience. But to justify it by experience "is begging the question. For all inferences from experience suppose, as their foundation, that the future will resemble the past, and that similar powers will be conjoined with similar sensible qualities" (ibid., p. 37). It "is impossible, therefore, that any arguments from experience can prove this resemblance of the past to the future; since all these arguments are founded on the supposition of that resemblance" (ibid., p. 38). If one claims that an argument from experience can prove it, then he "must be evidently going in a circle" (ibid., p. 36).

Thus Hume argues that induction might be justified only inductively, but such justification is impossible because the purported justification, being itself inductive, would be circular.

From this it appears that Hume's argument about induction is simply a special case of Sextus Empiricus's argument about the criterion of truth. As Weintraub points out, "Hume wasn't the first to cast doubt on the form of inference we call inductive," neither "was his argument original, initial appearances notwithstanding" (Weintraub 1997, p. 66).

Even the terminology concerning induction is due to Sextus Empiricus rather than to Hume. For, while Sextus Empiricus uses the term 'induction' [*epagoge*], Hume uses the terms 'by experience,' or 'experimental,' or 'causal,' rather than 'induction,' to refer to inductive arguments.

Moreover, Sextus Empiricus's argument about the criterion of truth applies both to deductive and non-deductive rules, while Hume applies his argument only to induction.

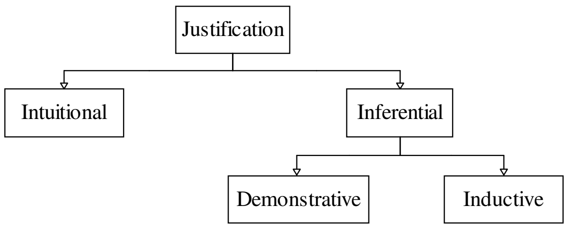
19. *Two Questions About Hume*

On account of this, the following two questions about Hume naturally present themselves:

1) Why did Hume apply his argument only to induction, refraining from extending it to deduction?

2) Since Hume's argument was not really original, what was his contribution?

As to 1), Hume distinguishes two kinds of justification: intuitional or inferential. Then he distinguishes two kinds of inferential justification: demonstrative or inductive.



Now, for Hume, no inferential justification of deduction is possible. For, on the one hand, no demonstrative justification of deduction is possible, since such justification would have to use those very same deductive rules that are being justified, and "the same principle cannot be both the cause and effect of another" (Hume 1978, p. 90). On the other hand, no inductive justification of deduction is possible, because induction itself cannot be justified.

Since, for Hume, no inferential justification of deduction is possible, and, on the other hand, he does not claim that deduction cannot be justified, we may conclude that he tacitly assumes an intuitional justification of deduction, namely, hat the validity of deductive rules is "discoverable at first sight" (ibid., p. 70). So the recognition of the validity of deductive rules falls "under the province of intuition" (ibid.).

Then the reason why Hume applies his argument only to induction is that he tacitly assumes that deduction has an intuitional justification.

This assumption has been widely shared since Hume.

For example, Gödel states that, for (MP), "there exists no other rational" justification except that it "can directly be perceived to be true," because "the proposition stating" (MP) is "an immediate datum" (Gödel 1986–2002, III, pp. 346-7 and footnote 34).

Similarly, Kyburg states that "our justification of deductive rules must ultimately rest, in part, on an element of deductive intuition: we see" that (MP) "is truth preserving – this is simply the same as to reflect on it and fail to see how it can lead astray" (Kyburg 1965, p. 276).

The assumption in question, however, does not appear to be plausible. The intuitional justification of deduction is inadequate for the very same reason why the intuitional justification of induction is inadequate. Namely, intuition is fallible, and, if one replaces infallible intuition by fallible intuition, then the justification of deductive rules ultimately depends on plausibility, and plausibility is based upon argument, not upon intuition.

As to 2), Hume's contribution was the asymmetry view. According to him, while deduction has an intuitional justification, the ground for an inductive inference "is not intuitive" (Hume 1975, p. 37).

Hume has strongly influenced the thought about induction, but his contribution does not consist in his argument about induction, which, as we have seen, is a special case of Sextus Empiricus's argument about the criterion of truth. It consists rather in the asymmetry view: deductive rules can be justified, while non-deductive rules cannot be justified.

The asymmetry view, however, is unjustified because, as we have seen, the justification of non-deductive rules raises problems similar to those of the justification of deductive rules, and must be approached much in the same way.

But, if the asymmetry view is unjustified, there is no such thing as 'the problem of induction,' because then there is nothing distinctively perplexing about induction as opposed to deduction.

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